RESIDUAL POROSITY OF A MEDIUM AFTER AN UNDERGROUND EXPLOSION

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Among the various practical uses of underground explosions, one of the most important is the use of such explosions to alter the permeability and porosity of the medium in the vicinity of the explosion. In light of this, it is necessary to theoretically calculate the residual porosity of the medium after an underground explosion has taken place. We are interested both in the spatial distribution of residual porosity and in the total volume of the pores created by the explosion.

It is possible to distinguish three main reasons for the change in the porosity of a medium after an explosion: 1) consolidation of the medium at the front of the shock wave; 2) loosening (or compaction) of the fragmented medium during its motion (dilatation effect); 3) deformation under the influence of the residual stress field. Also, new porosity develops in the region where radial cracks are located, due to the opening of these cracks. The goal of the present study is to analyze residual porosity in the fracture zone, where the above three mechanisms predominate.

Residual porosity was calculated theoretically in [1, 2] with the assumption of constant compaction at the front, a constant dilatation rate, and negligible elastic strains. Here, we analyze changes in porosity due to an explosion in different media.

1. We will examine a model of deformation of a granular, cemented medium saturated with moisture or gas. It is assumed that the initial medium consists of hard granules which are cemented together, with the intervening pore spaces being filled by fluid (gas, liquid). As an internal parameter of the model, we introduce structural porosity m_0 − the porosity of the medium without a load. The elastic strains are clearly nonlinear in character, due to the contact interaction of the grains (Hertz problem). The dependence of the running porosity m on the structure porosity m_0 is taken in the form

$$m = m_0/(1 + c\tilde{p}^n),\ \tilde{p} = (p_s - p_f)/K_s, \tag{1.1}$$

where K_s is the compressive bulk modulus of the solid phase; p_s and p_f are the pressures in the solid phase and the fluid. We can use Eq. (1.1) to obtain an equation for the volumetric deformation ε_d of an unsaturated brittle medium

$$-\varepsilon_d = \frac{m_0 c\tilde{p}^n}{1 + c\tilde{p}^n - m_0} + \frac{1 + c\tilde{p}^n}{1 + c\tilde{p}^n - m_0}(1 - m_0)\tilde{p},\quad \tilde{p} = \frac{p_s}{K_s}. \tag{1.2}$$

The constant c = 50 was determined from an analysis of the experimental curves $\varepsilon_d(p)$ for dry rocks with different initial porosities ($0 < m_0 \le 35\%$) [3]. The nonlinearity index n = 2/3 follows from the solution of the Hertz problem [4].

In the region of elastic deformation, the stress changes are connected with the strain rates by Hooke's law for a saturated nonlinearly elastic brittle medium:

$$\frac{d\tau}{dt} = G(\tilde{p}, m_0)\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right),\quad \frac{dp}{dt} = -K(\tilde{p}, m_0)\left(\frac{\partial u}{\partial r} + 2\frac{u}{r}\right) \tag{1.3}$$

($d/dt = \partial/\partial t + u\partial/\partial r$). With allowance for contact compressibility, the compressive bulk modulus K and the shear modulus G have the form

$$K(\tilde{p}, m_0) = \frac{K_s}{1 + \dfrac{m_0(1 - m_0)nc\tilde{p}^{n-1}}{(1 - m_0 + c\tilde{p}^n)^2}},$$

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$$G(\tilde{p}, m_0) = \frac{G_s}{1 + \frac{\xi m_0 (1 - m_0) n c \tilde{p}^{n-1}}{(1 - m_0 + c\tilde{p}^n)^2} \frac{G_s}{K_s}}. \tag{1.4}$$

Here, $1/\xi = 3(1 - 2\nu_0)/[2(1 + \nu_0)]$; $\nu_0 = \nu(\tilde{p}, m_0)|_{\tilde{p}=0}$ is the Poisson's ratio.

The fractured brittle material in the medium undergoes plastic strains if the plastic flow condition is satisfied. For the spherically symmetric case, we will take this condition in the von Mises–Huber–Schliecher form, with allowance for Tertsagi's law:

$$\frac{2}{\sqrt{3}}|\tau| = \alpha(\Lambda)(p - p_f) + Y, \tag{1.5}$$

where $p - p_f = (1 - m)(p_s - p_f)$; $\alpha(\Lambda)$ is the friction coefficient. It depends on the dilatation rate Λ and was obtained in [5]. The quantity Y represents adhesion.

The plastic flow is accompanied by an irreversible change in the volume (restructuring) of the fractured brittle medium – the dilatation effect. This effect is determined by the change in structural porosity m_0:

$$\frac{dm_0}{dt} = (1 - m_0)\Lambda(p_{ef}, m)\left|\frac{d\gamma^p}{dt}\right| \quad \left(\frac{d\gamma^p}{dt} = \frac{\partial u}{\partial r} - \frac{u}{r} - \frac{1}{G(\tilde{p}, m_0)}\frac{d\tau}{dt}\right). \tag{1.6}$$

For an unsaturated, low-porosity medium at low pressures, Eq. (1.6) becomes the familiar dilatation equation for a slightly compressible dilating medium [6]. The expression for dilatation rate $\Lambda(p_{ef}, m)$ is taken in a form similar to [7]:

$$\Lambda(p_{ef}, m) = \Lambda_0(m_*(p_{ef}) - m)/(1 - m) \tag{1.7}$$

($m_*(p_{ef})$ is the critical porosity. The expression for critical porosity was presented in [7]).

The total pressure p and density ρ are connected with the densities and pressures in the components by the relations

$$p = p_s(1 - m) + p_f m, \quad \rho = \rho_s(1 - m) + \rho_f m. \tag{1.8}$$

The equations of state of each phase are written in the form

$$p_s = \frac{\rho_{si} c_s^2}{\gamma_s}\left[\left(\frac{\rho_s}{\rho_{si}}\right)^{\gamma_s} - 1\right] \quad \text{for solid components,}$$
$$p_\ell = \frac{\rho_{\ell i} c_\ell^2}{\gamma_\ell}\left[\left(\frac{\rho_\ell}{\rho_{\ell i}}\right)^{\gamma_\ell} - 1\right] \quad \text{for liquids,} \tag{1.9}$$
$$p_g = p_{gi}\left(\frac{\rho_g}{\rho_{gi}}\right)^{\gamma} \quad \text{for gases.}$$

Here, ρ_{si}, $\rho_{\ell i}$, and ρ_{gi} are the initial densities of the components; c_s and c_ℓ are the speeds of sound in the solid phase and the liquid; γ_s, γ_ℓ, and γ are the adiabatic exponents of the solid, liquid, and gas.

As the closing equation, it is necessary to write a condition which determines the character of filtration of the fluid through the solid component. If we ignore filtration during the explosion, then we should have a constant value

$$m\rho_f/[(1 - m)\rho_s] = \text{const}, \tag{1.10}$$

which characterizes the ratio of the mass of the fluid to the mass of the solid in a unit volume.

In calculations of an underground explosion in [7], the chosen source of explosion-induced motion was a cavity with an initial radius a_0 filled with explosive gases. The pressure in the cavity changes in accordance with the adiabatic law

$$p(a) = p_0(a_0/a)^{3\gamma}, \tag{1.11}$$

where p_0 is the initial pressure of the gases in the cavity; p(a) is the running pressure; a is the running radius of the explosion cavity; γ is the adiabatic exponent of the explosive gases. The background pressure in the medium is equal to p_∞. The behavior of the brittle medium during the explosion is described by the equations of motion

$$\rho(\partial u/\partial t + u\partial u/\partial r) = \partial \sigma_r/\partial r + 2(\sigma_r - \sigma_\varphi)/r \qquad (1.12)$$

and continuity

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + 2\frac{u}{r}\right) = 0. \qquad (1.13)$$

Here, u is the mass velocity of the medium; ρ is density; r is the Eulerian coordinate; σ_r and σ_φ are the radial and azimuthal components of the stress tensor.

To perform numerical calculations, system (1.1)-(1.13) was written in finite-difference form, similar to [8]. We took the following values for the parameters in the calculation: K_s = 52 GPa, n = 0.67, c = 50, G_s = 24 GPa, ν_0 = 0.24, Y = 150 Pa, Λ_0 = 0.5, ρ_{si} = 2.65 g/cm³, $\rho_{\ell i}$ = 1 g/cm³, ρ_{gi} = 0.0012 g/cm³, c_s = 4500 m/sec, c_ℓ = 1600 m/sec, γ_ℓ = 6.3, γ = 1.4, γ_s = 7, p_∞ = 20 MPa, $\bar{p}_0$ = 70 GPa.

The results of numerical calculation of the residual change in the running value Δm of porosity as a function of distance are shown in Fig. 1 (gas-saturated medium) and Fig. 2 (water-saturated medium); W is the energy of the explosion. Curves 1-3 in Fig. 1 correspond to a gas-saturated medium with m_0 = 5, 15, and 35%, while curves 1 and 2 in Fig. 2 correspond to a water-saturated medium with m_0 = 5 and 35%. It is evident that three types of residual volumetric strains are possible in the vicinity of the explosion cavity after an underground explosion in a saturated, brittle-fracture medium: residual loosening (curves 1 in Figs. 1 and 2); residual consolidation (curve 3 in Fig. 1, curve 2 in Fig. 2); a nonmonotonic relation (loosening near the cavity, with subsequent consolidation going away from the cavity — curve 2 in Fig. 1). Residual loosening (0 < m_0 < 15%), nonmonotonic behavior (15% $\leqslant m_0$ < 30%), and residual consolidation (m_0 > 30%) are seen in the gas-saturated medium. In the water-saturated medium, residual loosening is seen up to m_0 ~ 25%. The path of porosity is slightly nonmonotonic at 25% < $m_0 \lesssim$ 30%, while residual consolidation — clearly seen from the numerical estimates — begins with $m_0 \geqslant$ 35%.

It should be noted that the given model is most suited for ideally brittle media, in which the change in porosity is connected mainly with the rearrangement of lumps of fractured rock and with the residual stress field. Here, no consideration is given to the possibility of irreversible strains connected with the potential flow of material into pores under the influence of high pressures in the shock front.

2. To determine the role played by irreversible consolidation at the front in terms of the effect of such consolidation on residual porosity, we will examine a model of a medium in which consolidation at the front decreases with distance from the center of the explosion, while the strains of the medium behind the front are characterized by a constant dilatation rate Λ [9].

Following [9], we assume that the source of motion of the medium is a gas located in a cavity with an initial radius a_0. At t > 0, a spherical shock wave begins to propagate from the cavity. At the front of the shock, the medium is instantaneously compacted as a result of collapse of pores. The degree of compression of the medium at the front will be characterized by the consolidation ε(R) = 1 − ρ_0/ρ(R), where R is the radius of the shock-wave front and ρ(R) is the density attained at the front. A similar law of change in consolidation was used in [10]. It is assumed that the medium undergoes fracture immediately after consolidation. Behind the shock front — which coincides with the front of the fracture wave — the medium undergoes plastic flow accompanied by a density change, due to the dilatation effect [1, 2]. Here, no allowance is made for the compressibility of the lumps of fractured medium. The medium is described by equations of motion and continuity and the dilatation equation, similar to [6]. The dilatation rate Λ is assumed to be constant. It is also assumed that the Prandtl plasticity condition $\sigma_r - \sigma_\varphi = k + m_1(\sigma_r + 2\sigma_\varphi)$ is satisfied behind the wave front (k and m_1 are the adhesion and friction coefficients). With allowance for the laws of conservation at the shock front and the adiabatic nature of expansion of the explosive gases in the cavity for the relation $\varepsilon(R) = \varepsilon_0(a_0/R)^\lambda$ $(\lambda > 0)$, we have an equation for the medium which, when solved, can give us a complete description of the motion of the medium within the framework of the following assumptions:

$$\frac{dy}{dx} + N(x)\,y = M(x). \qquad (2.1)$$

Here, $y = \dot{x}^2$; $\ddot{x} = \frac{1}{2}\frac{dy}{dx}$;

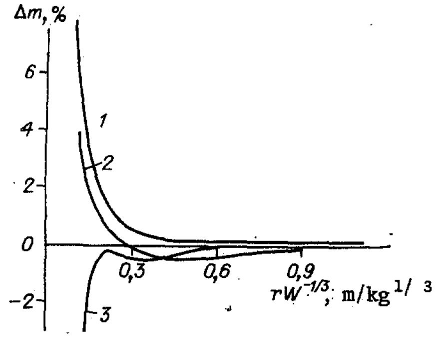

Fig. 1

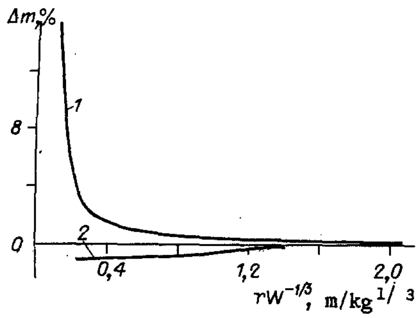

Fig. 2

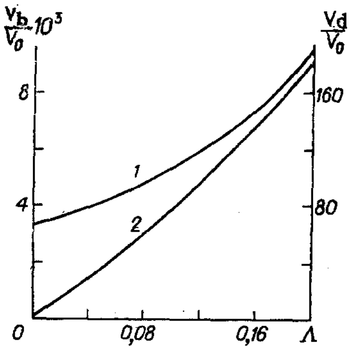

Fig. 3

$$M(x) = 2\frac{[Z^{\alpha}(t) - x^{\alpha}]\frac{k}{3m_1 p_0} - x^{\alpha}\left[\Sigma_r(x) - Z^{\alpha}\frac{\sigma^*}{p_0}\right]}{Y_0 x^n};$$

$$N(x) = \frac{2n}{x} - 2\frac{x^n}{Y_0}\left[nX_0 - \varepsilon(Z) Z^{\alpha-2(n-\lambda)}\right];$$

$$X_0 = \int_1^Z r_0^2 r^{\alpha-3-2n}(r_0)\,dr_0; \quad Y_0 = \int_1^Z r_0^2 r^{\alpha-2-n}(r_0)\,dr_0;$$

$$\alpha = 6m_1/(2m_1+1);\ n = (2-\Lambda)/(1+\Lambda);\ x = a/a_0;\ Z = R/a_0;$$

$$\Sigma_r = \sigma_r/p_0;$$

σ^* is the crushing strength of the medium; $r(r_0)$ is the dependence of the Eulerian coordinate on the Lagrangian coordinate, which is found from the equation $\frac{\partial r}{\partial r_0} = \frac{1}{\rho}\left(\frac{r_0}{r}\right)^2$; $\rho(r) = \frac{1}{1-\varepsilon_0 r_0^{-\lambda}} \times \left(\frac{r_0}{r}\right)^{2-n}$; r and r_0 are expressed in the units of a_0, while ρ is expressed in the units of ρ_0; the dot denotes differentiation with respect to the dimensionless time $\tau = t\sqrt{p_0/\rho_0 a_0}$. The initial condition for (2.1) has the form $y(x = 1) = \varepsilon_0$. The solution of the equation of the medium gives the time dependence of the radius of the explosion cavity and is unambiguously connected with it by the law of mass conservation for the radius of the fracture-wave front. Ignoring oscillations of the walls of the cavity about their final position, we will assume that the maximum values of the radii of the cavity and the fracture-wave front are also the final values of these quantities.

Now let us proceed to the derivation of formulas that will allow us to calculate the volume of the pore space using very general assumptions on the behavior of the medium and its parameters. We assume that the explosion-induced motion of the medium is described by the model presented above. As already noted, the irreversible compaction of the medium is connected with the partial collapse of pores at the shock front. The residual porosity of the blocks into which the medium has fractured will not change in the subsequent flow behind the front. We will henceforth refer to this porosity as the intra-block porosity, and we will refer to the porosity which results from the dilatation effect as the dilatational porosity. At an arbitrary moment of time, the total volume of the pores which comprise the intra-block and dilatational porosities is equal to $V = \int m(r, t)dV$ [$m(r, t)$ is the total porosity of the medium at point r at the moment of time t]. The integral is taken over the entire volume of the medium involved in motion. The expression for m has the form $m(r, t) = 1 - (1 - m_u) \times \rho(r, t)/\rho_0$ (m_u is the porosity of the undisturbed medium).

Then, with allowance for the sphericity of the motion, we find that

$$V = \int_a^R \left[1 - \frac{(1-m_u)}{\rho_0}\rho(r, t)\right] r^2 dr. \tag{2.2}$$

Integration of (2.2) with allowance for the mass conservation law leads to the expression

$$V = \frac{4\pi}{3}(R^3 - a^3) - \frac{4\pi}{3}(1-m_u)(R^3 - a_0^3). \tag{2.3}$$

We divide both sides of (2.3) by the initial volume of the explosion cavity V_0. Then, in dimensionless form, we obtain

$$\tilde{V} = m_u(\tilde{R}^3 - 1) - \tilde{a}^3 + 1, \ \tilde{V} = V/V_0, \ \tilde{a} = a/a_0, \ \tilde{R} = R/a_0. \quad (2.4)$$

Equation (2.4) makes it possible to calculate the total volume of pores in the medium at any moment of time, including the moment when the cavity stops moving. It should be noted that the function $\tilde{V}$ was obtained with an arbitrary law for consolidation of the medium at the shock front, and it does not explicitly contain the dilatation rate.

Having determined m_u, a_m, and R_m from experiment (a_m and R_m are the maximum dimensions of the cavity and the fracture zone), by using (2.4) we can easily find the total volume of pores in the medium after an underground explosion.

Now let us calculate the volumes of the pores associated separately with intra-block and dilatational porosity. For the volume associated with intra-block porosity, we have

$$V_b = 4\pi \int_{a_0}^{R} [m_u - \varepsilon(r_0)]\, r_0^2 dr_0$$

[$\varepsilon(r_0)$ is the consolidation of the medium at the front at the Lagrangian point r_0]. With the power relation $\varepsilon(r_0) \sim r_0^{-\lambda}$, we obtain the following for the dimensionless total volume of the intra-block pores

$$\tilde{V}_b = m_u(\tilde{R}^3 - 1) - \frac{3\varepsilon_0}{3-\lambda}(\tilde{R}^{3-\lambda} - 1) \quad (\lambda \neq 3). \quad (2.5)$$

The total volume of inter-block cavities is obtained by integrating the expression for dilatational porosity $m_d(r_0, t) = 1 - \left[\frac{r_0}{r(r_0, t)}\right]^{2-n}$ over the volume of the fractured medium. Finally,

$$\tilde{V}_d = \frac{3\varepsilon_0}{3-\lambda}(\tilde{R}^{3-\lambda} - 1) - \tilde{a}^3 + 1 \quad (\lambda \neq 3). \quad (2.6)$$

It is easy to see that the sum of (2.5) and (2.6) leads to Eq. (2.4). Thus, Eqs. (2.4)-(2.6) make it possible to determine both the total volume of pores and the volumes connected with intra-block and dilatational porosity, respectively. Meanwhile, to determine the total volume of pores, it is necessary to determine only m_u, R, and a experimentally. The parameter λ can be found from data on the shock compressibility of the medium, knowing the law by which the peak stresses at the front of the fracture wave decay over the radius.

Figures 3 and 4 show the results of calculations with Eqs. (2.5) and (2.6), where $\tilde{m}_u$ = 6%, $\tilde{m}_1$ = 0.2, k = 20 MPa, ε_0 = 0.06, σ^* = 30 MPa, Λ = 0.07. The dimensions of the cavity and the fracture zone were obtained from the solution of the equation of the medium (2.1). Figure 3 shows the dependence of the total volume of the residual intra-block and inter-block pores on the dilatation rate Λ (lines 1 and 2). As is known, an increase in dilatation rate leads to an increase in the degree of loosening of the medium in the fracture zone and, thus, to an increase in the total volume of the dilatational pores. Displacement of the medium from the region in which dilatational loosening takes place causes an increase in the radius of the fracture zone and a decrease in the final radius of the cavity (see [11, 12]). In accordance with Eq. (2.5) for the total volume of intra-block pores, this leads to an increase in the latter — as is confirmed by the path of curve 1 in Fig. 3.

Figure 4 shows the dependence of the volumes of dilatational and intra-block cavities on the consolidation index λ (the notation is the same as in Fig. 3). The increase in V_b with λ has a trivial explanation: the more rapid the decrease in the consolidation of the medium at the front, the higher the value of residual porosity of the blocks into which the medium fractures. There is also an increase in the dimensions of the fracture zone [11], so that an increase in the consolidation index λ leads to a sharp increase in the total volume of the residual intra-block pores. The relation $V_d(\lambda)$ is more complicated. As noted above, an increase in λ is accompanied by a decrease in the radius of the explosion cavity and an increase in the size of the fracture zone. Thus, expansion of the region affected by the explosion leads to an increase in the volume subjected to dilatational loosening. On the other hand, with a reduction in the compression of the medium at the wave front, there is a reduction in the amount of material displaced as a result of pore collapse. The reduction in the displacement of the medium and the associated reduction in shear strain decrease the amount of dilatational loosening which occurs. The competition among these effects leads to the nonmonotonic relation $V_d(\lambda)$ shown in Fig. 4.

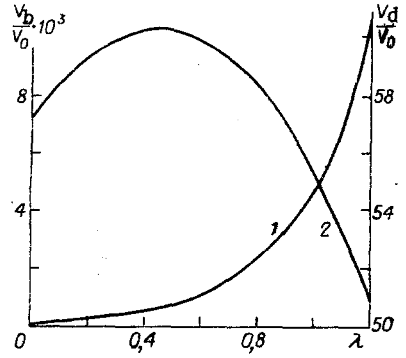

Fig. 4

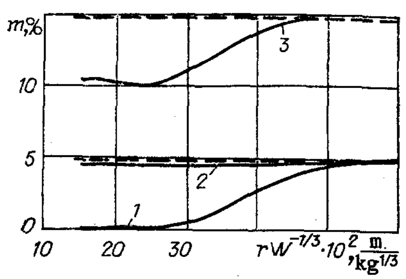

Fig. 5

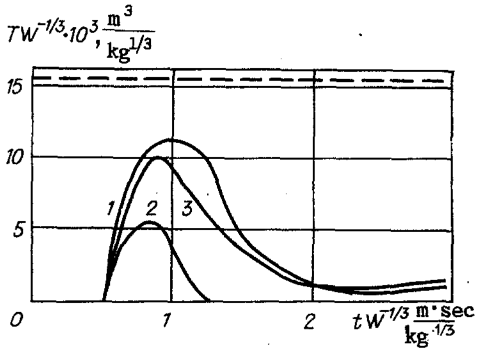

Fig. 6

3. To determine the residual porosity of a plastically deformed medium, we can use the method in [13] to perform numerical calculations. The authors of [13] used the model equation state in [14] (similar to the equation proposed in [15]) to describe the behavior of the porous medium. In accordance with this equation, the behavior of a porous medium with volumetric strains is irreversible in character. The degree of irreversible compaction depends appreciably on the type of substance saturating the pores – gas or liquid. In the first case, porosity decreases to almost zero during fracture and pore compression. In the case of saturation by a liquid, there is little change in porosity, due to the low degree of compressibility of the liquid. Intermediate cases will exist for a combination gas–liquid medium.

Figure 5 shows the results of numerical calculations of the dependence of residual porosity on distance. Curve 1 corresponds to a gas-saturated medium with an initial porosity of 5%, curve 2 corresponds to a water-saturated medium with the same initial porosity, and curve 3 corresponds to a combination gas–water-saturated medium with an initial porosity of 15% – 5% for the gas phase and 10% for the liquid phase. The dashed lines show the background porosity. The curves confirm the above-described features of irreversible volumetric deformation of a porous medium. It should be noted that, in the case of a medium with gas, the dimensions of the regions where gas porosity is completely and partially eliminated are roughly the same.

One important mechanical effect of an underground explosion is the formation of a zone of brittle radial cracks [16]. The formation of such a zone in monolithic rock was considered in the numerical calculations, and the total volume of cavities in this zone was determined. The formation of a zone of radial cracks was accounted for in accordance with the following scheme. When the azimuthal radial stresses in a Lagrangian particle of the medium exceed the cohesive strength σ_0, a brittle radial crack having the same size as the particle is formed; in the particle itself, there is a sudden change in density, the stresses σ_r and σ_φ, and pressure. These sudden changes are easily calculated from the condition of continuity of the radial component of the strain tensor: $[\sigma_\varphi] = -\sigma_0$, $[\sigma_r] = -2\nu\sigma_0$, $[\tau] = (1 - 2\nu)\sigma_0$, $[p] = (1+\nu)\sigma_0 2/3$, $[V] = -(1+\nu)V\sigma_0/K$. Here, the symbol [ ] denotes a sudden change in a quantity, i.e., the difference between the new and old (prior to fracture) values of the corresponding quantity.

With the formation of cracks as a result of stress relief, a certain volume of cavities T(t) is created. This volume can be identified with the volume of the cracks and calculated from the formula $T(t) = 4\pi \int_\Omega \left(1 - \frac{V_T}{V}\right) r^2\, dr$, where V_s is the specific volume of the solid phase between the cracks and V is the total specific volume of the medium with cracks. The integral is taken over the region in which cracks are present Ω.

Figure 6 shows the relation T(t) for different cases of background pressure p_∞ in a medium and the cohesive strength. Curves 1-3 corespond to p_∞ = 2, 5, and 2 MPa and σ_0 = 3, 3, and 5 MPa, respectively. The other parameters of the medium had the following values: K = 54 GPa, ρ_0 = 2.7 g/cm³, ν = 0.33. As the numerical calculations showed, the volume of the pores created is quite large and is comparable in terms of dynamics with the final volume of the explosion cavity (the dashed line in Fig. 6). However, as a result of the reverse motion of the medium in the vicinity of the cavity, the radial cracks may be partially or completely closed and their total volume may decrease by nearly one order of magnitude – it may even reach zero (curve 2).

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